**Можно написать, что мы исследуем такую связь и видим, что получается простая формулировка (в одном случае совпадающая с предыдущими)**

**A geometric approach to blob detection in color images**

**(A Riemahniann approach to blob detection in color images) (x)**

**Riemahniann blob detection**

**Abstract**: Full use of color information has a big influence on quality of keypoints detection in color images. The more color information we use, the better we can distinguish features, and the more stable detection is. Most methods of keypoints detection in color images utilize in some way a conversion of image to grayscale, what results in loss of some information contained in color.

In this paper we propose a new generalization for color images of blob detection, a widely used keypoints detection method. Contrarily to the previous approaches we don’t use any kind of conversion to grayscale. We consider color image surface as a smooth manifold and reformulate original method through the scalar curvature of image manifold and domain manifold. This formulation provides a straightforward (and natural) generalization for color images. The generality of analysis makes our method applicable to a broad range of problems and data models. The effectiveness (and superiority) of the proposed method is shown by the experiments in chemical compounds classification task. The algorithm is easy to implement and as fast as original method.

1. **Introduction**

Keypoints detection is applied in a wide range of tasks such that image mosaicing, image registration, panorama stitching, 3D recovery, tracking, recognition, shapes modeling, etc. Its usage is not limited to planar images, it is used also for 2d-surfaces[], volumetric data[], video data[], etc. Keypoints are usually (seeked) found as low-level features, likely to be stable under some transformations.

Blob detection [, , , , ,] is a widely used keypoints detection method. Informally speaking, it aims to find ellipse-like regions of different sizes in grayscale image with “similar” intensity inside. It is the base of widely used SIFT [] and SURF [] features extraction methods. It can be applied not only to images on Euclidian domain, but also to scalar functions on manifolds[, , ,]. It has applications for different problems with different kinds of data - 3D face recognition, object recognition, panorama stitching, 3D scene modeling[], tracking[], action recognition[], medical images processing[].

For color images full use of color information has a big impact on quality of keypoints detection. The more color information we use, the better we can distinguish features, and the more stable detection is. As to our knowledge all previous color blob detection methods are based on global [,,,] or local [,,,] conversion to grayscale. We propose another approach, which doesn’t use any kind of conversion to grayscale. It is based on reformulation of original method through the scalar curvatures of image manifold and domain manifold. This formulation provides a straightforward and (natural) generalization of blob detection for color images.

Establishing a connection between image processing methods and the geometry of image surface is of its own interest. It helps deeper understand traditional methods[,], provides insights[,] and gives natural extensions of classical methods to color images[,]. The connection between blob detection and image surface curvature was mentioned in several papers [, ,], but there were errors in proposed analysis. So our paper is the first to accurately analyze this question in the general setting.

Recently there is a big demand in processing data (naturally) lying on manifolds. In informational geometry [, ,], proteins modeling [, ,], chemical compounds classification[, ,], 3d reconstruction [,], 3d models recognition [], action recognition we deal with functions on the manifolds. Because of this, methods, formulated in a general setting of non-Euclidian domain, are needed. All previous color blob detection approaches were defined in a narrow setting – for image in 2D Euclidian domain. In contrast, we consider image as a smooth vector-valued function over n-dimensional manifold. So our method is applicable to a broad set of problems and data representations.

It is important to notice, that speed of computation is a crucial aspect for many image processing applications. A large amount of research was devoted to make blob detection faster [, , ,] by using different approximations, sampling strategies, etc. Our method is easily implemented on the base of grayscale blob detection, so each fast implementation of blob detection can be extended to obtain the implementation of our method. Our method is as fast as grayscale blob detection.

**Contributions:**

We present a generalization of blob detection method for color images, defined as vector-valued function over smooth n-dimensional manifold. This method is based on reformulation of original method through scalar curvatures of image surface and domain manifold.

1. We are the first, to our knowledge, to present blob detection in color images without usage of global or local conversion to grayscale. The algorithm is as fast as grayscale blob detection and easy to implement on the base of original method.
2. We are the first, to our knowledge, to analyze a connection between blob detection method and scalar curvatures of image surface and the domain manifold.
3. We are the first to propose and analyse color blob detection for a general case of a m-dimensional vector function over smooth n-dimensional manifold. So our method applicable to a broad set of problems and data representations.
4. We provide experimental results of our method application to the task of chemical compounds classification. The results show the effectiveness (and superiority) of the proposed approach.

The remainder of the paper is organized as follows. In section 2 we review related work. In section 3 we present our main results – color blob detection through scalar curvatures of image surface and domain manifold (with theoretical analysis), and the algorithm for its computation on the base of grayscale blob detection algorithm. In section 4 we propose the results of our method application to the task of chemical compounds classification. In section 5 we give conclusions and discuss possible directions of the future research.

1. **Related work**

**Blob detection**

Blob detection was firstly proposed in works [, , , ,] for images on 2D Euclidian domain. Different versions of this method were used as a part of well-known SIFT [] and SURF [] algorithms. In works [, ,] blob detection was generalized (in different ways) for finding keypoints locations and sizes on 2D surfaces embedded in and represented by triangular mesh.

**Color blob detection**

Some approaches were proposed to adapt blob detection for color images on 2D Euclidian domain. These approaches are based on global or local conversion of image to grayscale. In [] processed image is converted to grayscale by projecting color onto vector, obtained by applying PCA to color vectors. In [] authors propose local projection on vector function obtained by applying Laplacian to each image channel. In [] authors propose local projection on vector function which is found by maximizing “blobness” of projection.

**Image processing by (based on) image surface geometry**

Geometry of image surface was used in different image processing methods. In [] metric of image surface was used for generalization of gradient detection to color images. In [, ,] the framework was proposed which allows to reformulate many diffusion methods as Polyakov action on image manifold. In [, ,] the applications of discrete Forman curvatures to image processing were presented. In [, ,, ] image was considered as a section of trivial Clifford bundle over smooth manifold which allows for generalized definitions of gradient, diffusion, Fourier transform.

1. **Proposed method**

Firstly we will give here the formal definition of the blob detection for scalar function on surface with usage of Hessian determinant as *feature response function*:

Suppose we have . Blob detection method is as follows:

1. Calculate the scale-space : the scaled solution of heat equation on surface

, where is a Laplace-Beltrami operator on surface.

1. Rescale
2. Calculate feature response function where is a Hessian of with fixed .
3. Find blobs centers as . Find blobs radius as ;

**Color blob detection**

Let’s consider the case of vector-valued image . The generalization of steps 1 and 2 (scale-space construction) to the color images is straightforward because of the linearity of the heat equation. Then look at the step 3. For the vector-valued function its Hessian is a covariant differential of the function differential []: , so we can’t use as a feature response function.

How can this problem be solved without conversion of to scalar function? The key observations in our solution were the following:

1. We can consider image as a 2-dimensional manifold embedded in . Then grayscale and color cases differ only by co-dimension of embedding. So if we reformulate feature response function through some intrinsic manifold characteristics, it will not depend on and thus it will be defined both for grayscale and color cases.
2. How can we reformulate feature response function through intrinsic manifold characteristics? The hint is the following: if image surface tangent plane is close to the plane then scalar curvature of image surface (which is intrinsic) is close to the determinant of image Hessian.

**Proposed method**

Consider a smooth connected manifold (isometricaly embedded in ). Denote metric on it and associated Levi-Civita connection.

There is a vector-valued function . Denote a trivial vector bundle with base manifold . is a smooth (n+m)-dimensional manifold.

Let be a section of . Let be an image of , i.e. . is a smooth n-dimensional manifold, embedded in by definition.

Let . is embedded in by .

Our generalization (main result) follows from the following theorems:

**Theorem 1**: Let is a smooth function, is a smooth 2D manifold. and are scale-space and feature response functions respectively (see p.1)

Then ,

where is a scalar curvature of a manifold.

Theorem 1 immediately gives the feature response function for color images: all the used notions are intrinsic and thus absolutely the same expression can be written for color case.

So it gives a natural definition of the generalized feature response function:

**Definition 1:** Let is a smooth function, is a smooth n-dimensional manifold

Theorem 2 provides a usual formulation of feature response function through the image Hessian:

**Theorem 2**: ,

where , is a Hessian of with fixed t,

and is the orthonormal basis of .

Theorem 2 gives a gives a way for straightforward implementation of our method on the base on grayscale blob detection. Suppose we have some algorithm for grayscale blob detection, which approximates feature response function. Then we can obtain from per-channel . This process is given in Algorithm 1.

**Algorithm 1.**

Input:

– image domain in some discretized representation (pixelized, triangulated, etc.)

- color image defined on image domain

– some algorithm which computes for a grayscale image

Algorithm:

1. Fix orthonormal basis in color domain

2. Denote . Compute .

3. Compute

**4. The experiments**

We apply our color blob detection method to the problem of chemical compounds classification. The task is to distinguish active and non-active compounds on the basis of their structure. It’s based on the hypothesis that compounds with similar structure should have similar properties. In the literature it is called the QSAR problem [].

In our experiments each compounds was represented by its molecular surface and physico-chemical properties, defined on the surface. The molecular surface is a 2D surface drawn around the atoms coordinates []. The shape of this surface and distribution of physico-chemical properties on it has a crucial influence on the compound activity []. So the input data can be modeled as a manifold with a vector-valued function on it , where coordinates of are different physico-chemical properties.

We used our approach for the construction of descriptor vectors. The procedure is the following:

1. Detect blobs by our method for each compound surface.
2. Form pairs of blobs on each surface
3. Transform blobs pairs into vectors of fixed length by using bag of words approach [].

The details of the experiments are the following. Molecular surfaces are represented by triangulated surfaces. We use the following properties: electrostatic potential, Lennard-Johnson potential [], also geometrical properties – Gaussian curvature and mean curvature. These properties are defined in each node of triangulation. We use algorithm [] for Gaussian and mean curvatures calculation.

The implementation of blob detection method is the following:

1. We calculate scale-space by usage of iterated approximation of the heat equation solution [].
2. We found Hessian as covariant differential of function differential. In detail,
   1. We have
   2. We find functions directional derivatives by finite elements differences, where are directions from central vertice to neighbours.
   3. We find differential by solving overdefined linear equation , is a matrix which columns are vectors
   4. We find covariant derivatives of the differential in neighbor directions, i.e. find for each as by . are found by finite elements differences.
   5. We find covariant differential by solving overdefined linear equation , is a matrix which columns are vectors
3. is obtained. We calculate .
4. Find blobs centers as . Find blobs radius as

We compared four methods:

1. Our method;
2. A naïve method of applying blob detection to each channel separately;
3. Method of adaptive projection from []. It is adapted by us to the case of 2Dsurface;
4. Method of adaptive neighbourhood projection []. It is adapted by us to the case of 2Dsurface.

We used cross-validation functional [] as an index of performance quality. The data for test was the following: 3 datasets (bzr, er\_lit, cox2) were taken from [], 3 datasets (glik, pirim, sesq) were taken from Russian.. . The results are present in the table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | **naive** | Adaptive projection [] | Adapt. neighbor. projection [] |
| glik | 1.0 | 0.954 | 0.975 | 1.0 |
| pirim | 0.99 | 0.96 | 0.97 | 0.98 |
| sesq | 1.0 | 0.98 | 0.976 | 1.0 |
| bzr | 0.992 | 0.971 | 0.975 | 0.983 |
| er\_lit | 0.98 | 0.961 | 0.956 | 0.98 |
| cox2 | 0.991 | 0.967 | 0.985 | 0.986 |

We can see that our method outperforms all other methods half of datasets, on other half it performs the same as adaptive projection method. This shows the effectiveness and the superiority of our approach.

**6. Conclusion and future work**

We propose the method of blob detection in color images. Contrarily to the previous approaches we don’t use any kind of conversion to grayscale. The generality of analysis makes our method applicable to a broad range of problems and data models. The algorithm is easy to implement and as fast as original method. Our approach outperforms all other methods on the task of chemical compounds classification.

The directions for the future work can be the following:

1. Generalization of our method for non-trivial vector and tensor bundles (now it is defined only for trivial vector bundles). So that method will be applicable tangent bundles and others useful cases.
2. Generalization of our method for infinite-dimensional domain manifold. This is motivated by the recent popularity of such concepts as shape space []. The shape space is infinite-dimensional smooth manifold, so our method should be further generalized to be applicable to it.